Math 434 Assignment 1

Due March 15

Assignments will be collected in class.

2. We defined addition on the natural numbers. Using this definition, prove:

   (a) for all $a \in \mathbb{N}$, $0 + a = a + 0 = a$.

   (b) for all $a, b \in \mathbb{N}$, $a + b = b + a$.

   Solution: We argue by induction on a. We already know by part (a) that for $a = 0$, for all $b \in \mathbb{N}$ that $a + b = b + a$. Suppose that for a given a, for all $b \in \mathbb{N}$, $a + b = b + a$. We must prove that for all $b \in \mathbb{N}$, $s(a) + b = b + s(a)$.

   Fix a; we will now argue by induction on b. This is true for $b = 0$, as $s(a) + 0 = 0 + s(a)$. Supposing that $s(a) + b = b + s(a)$, we will argue that $s(a) + s(b) = s(b) + s(a)$. Indeed,

   $$s(a) + s(b) = s(s(a) + b) = s(b + s(a)) = s(s(b + a)).$$

   and

   $$s(b) + s(a) = s(s(b) + a) = s(a + s(b)) = s(s(a + b)) = s(s(b + a)).$$

   (Note that we use the induction hypothesis on b in the first line, and the induction hypothesis on a in the second.)